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Assessing Probabilities of Unique Events: A New Approach

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Decision-makers in unique or one-off situations may have difficulties in framing the probabilities of possible events that are required in modern decision-making. This paper illustrates a new approach to probability determination based on pairwise primary judgments on the relative likelihoods of the possible events. Related "news" on the situation can also be used to update these prior probabilities using Bayesian Revision. Illustrative calculations outline the entire process through to determination of posterior probabilities following a "news" event.

Keywords: Lehman failure, probabilities, odds, Analytic Hierarchy Process (AHP), Bayesian Revision

Introduction

The Lehman failure in 2008 that triggered the global financial crisis is a classic example of a situation involving probability assessment based only on judgment. At the time, PIMCO (a fixed income investment manager) postulated three possible scenarios for a Lehman future:

- A takeover by a stronger bank as occurred with J. P. Morgan taking over Bear Stearns;
- An orderly liquidation similar to that of Long Term Capital Management in 1998;
- A disorderly liquidation with asset sales at distress values.

In his book *The Only Game in Town*, Mohamed El-Erian (2016) stated the probabilities assigned to the scenarios were the highest for the takeover (in fact "relatively high") and the lowest for scenario three, which was the event that obtained. Assigning probabilities in these situations comes down to judgment on the part of the decision-maker (DM) since there is little or no "objective" data to utilize.

Assessing Probabilities Directly

Suppose the problem involves Event E and the assessment is the probability of E occurring or not occurring which is Event E^c. Then, only one primary judgment (PJ) is involved since P(E) also determines $P(E^c)$.

For three or more events, more than one PJ is involved and problems can arise. Suppose the events are A, B, and C with A least likely and C most likely in the decision-maker's judgment, these events are not related to the Lehman failure. Further suppose a direct assessment leads to P(A) = 0.15 and then P(B) = 0.25 which necessarily means P(C) = 0.6, that is, the PJs about Event C play no direct role in the determination of a 60% chance for this event other than its initial determination as the most likely event.

Now suppose that the DM thinks the 60% chance for Event C is too low and a 70% chance is the correct evaluation of the likelihood of C occurring, he/she now re-evaluates P(B) at 0.2 and therefore necessarily P(A) becomes a 10% chance, lower than his/her initial judgment of 15%.

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Recursion of this approach would lead to a probability distribution over all events that the DM ultimately finds acceptable. However, the problem with this direct approach is that some information on one event plays little or no role in determining the initial probability distribution. A new approach can supersede this roundabout procedure.

A New Approach

Using relative likelihood judgments (the PJs) rather than absolute judgments allows all information on all events to play a role in determining the initial probability distribution. Although these PJs may be rudimentary in nature as to the extent of what is "more likely", using even vague or imprecise numerical assessments enables the utilization of the full apparatus of decision theory based on the resulting probability distribution. The Analytic Hierarchy Process (AHP) developed by Thomas L. Saaty (2005) is based on pairwise comparisons of alternatives and provides the rationale for the assessment procedures outlined here. Initially, the events are ordered from least likely to most likely with successive numerical pairwise comparisons through to the most likely event.

Suppose P(B)/P(A) is evaluated as 2 meaning B is twice, as likely as A and P(C)/P(B) is evaluated at 3 meaning C has triple the likelihood of B, the PJs on all three events therefore play a role in determining the initial probability distribution as outlined in Table 1.

Table 1

Derivation of a Probability Distribution Based on "More Likely" Judgments on All Events

Events	Likelihood	Pairwise likelihood	Compound likelihood	Probability
A	Least likely event	1.00	1.00	1/9 = 0.11
B	Intermediate likelihood	2.00	1 × 2 = 2.00	2/9 = 0.22
C	Most likely event	3.00	1 × 2 × 3 = 6.00	6/9 = 0.67
			9.00	1.00

Note that the least likely event is standardized at 1.0 and that 0.22/0.11 = 2 as determined by the PJs initially. Given the rudimentary nature of the PJs, a two-decimal probability seems appropriate allowing customary percentage pronouncements on the probabilities so derived. An equally likely event in the ordering at any point would be denoted by a value of 1.0.

If we use the initial absolute primary judgments above, then we get P(B) /P(A) at 0.25/0.15 or 1.67 and P(C)/P(B) at 0.7/0.2 or 3.5. Using these pairwise values and the above methodology, the probabilities for A, B, and C become respectively 0.12, 0.2, and 0.68. Not too different from the probabilities derived in Table 1.

Tentative direct probability assessments, such as those posited initially could form the basis for the pairwise values to be used in the methodology outlined here even if collectively these values do not obey all the probability axioms. It is possible for the probability ratios to be more accurate than the probabilities themselves.

The "more likely" values used here may also be motivated by rough odds judgments. For example, a 9:1 against event is 10% "more likely" than a 10:1 event (as in eleven trials for one occurrence versus ten for the first event and hence 11/10) and 50% "more likely" than a 14:1 event (15/10). Odds judgments typically result from expert assessment (for repeatable or similar trials) whereas "more likely" values can be formulated by non-experts (for one-off situations) based only on intuition. And if this intuition can be developed over time then these formative "more likely" primary judgments could supplant the necessity for the ensuing but more demanding assessments of odds and/or probabilities in the first instance.

Note that a perfectly consistent reciprocal matrix for the judgments above can be formed using 6 in the C,A position (as in 2 × 3) and 1/6 in the A, C position (as in A has 1/6 of the likelihood of C). Priority values for A, B, and C are then determined as the geometric mean of each row and normalized to sum to unity as in a complete AHP analysis. The resulting priorities or probabilities for A, B, and C are exactly the same as in Table 1 due to the perfectly consistent reciprocal matrix. If a non-consistent 5 were the judgment in the C, A position (a reduction in C's relative likelihood over the previous 6 times more likely compared to A) and 1/5 in the A, C position, then the probabilities for A, B, and C respectively become 0.12, 0.23, and 0.65 reflecting a slight reduction in Event C's likelihood over the previous 0.67.

Saaty (2005) proposed deriving the probabilities (or priorities) using the principal eigenvector of the reciprocal matrix. The geometric mean method used here is an alternative that "is very appealing for practical applications" according to Brunelli (2015). In practice, Budescu, Zwick, and Rappoport (1986) compared both procedures and concluded that in most cases the solutions are very similar to each other.

Additional judgments between non-adjacent events may add more information that (if not radically inconsistent) could result in a probability distribution using a complete AHP analysis that is more accurate than that using only the adjacent events as above in Table 1.

In this illustrative example, Events A and B end up being slightly less likely than the initial absolute judgments above with Event C ending up between the initial and second probabilities. If one event is judged to be between 2 and 3 times more likely than another event, a pairwise value of 2.5 would be appropriate. Or a value closer to 2 or 3 could be used if the DM can make a more precise assessment.

Bayesian Revision

Suppose "news" arrives on this situation and the DM decides to revise his/her probability distribution in the light of this "news". Now, the relevant pairwise numerical judgment becomes "more consistent with the news" replacing "more likely" as above. Suppose that the order from least to most consistent with the "news" is C, A, and B. Illustrative pairwise judgments on the "news" and resulting likelihoods for each event given the "news" (i.e., the likelihood the given event is the true state of the world based only on the "news") are detailed in Table 2.

Table 2

Derivation of the Likelihood of Consistency of the Event With the "News"

Events	Consistency level	Pairwise likelihood	Compound likelihood	Likelihood of event
C	Least consistent	1.00	1.00	1/5.5 = 0.18
A	Intermediate consistency	1.50	1 × 1.50 = 1.50	1.5/5.5 = 0.27
B	Most consistent	2.00	1 × 1.50 × 2 = 3.00	3/5.5 = 0.55
			5.50	1.00

So, A is significantly rather than slightly more consistent with the "news" than is C (hence 50% or 1.5 and not 10% or 1.1) and B is twice as consistent as A. An equally consistent event in the ordering would be denoted by a value of 1.0.

In some cases, some events will have zero consistency with the "news" and can be eliminated altogether. For example, if it is learnt that the appointee to a position will be Asian, then all non-Asian candidates can be eliminated. In this case, the posterior probabilities for the remaining Asian candidates will be the renormalized prior probabilities since the "news" assigns the same likelihood to each Asian candidate and zero likelihood to all other candidates.

For the illustrative “more consistent” judgments in Table 2, the derivation of the posterior distribution after the “news” event is shown in Table 3.

Table 3

Derivation of the Posterior Distribution Following the “News”

Events	Prior	Likelihood	Joint	Posterior probability
A	0.11	0.27	0.11 × 0.27 = 0.0297	0.0297/0.2713 = 0.11
B	0.22	0.55	0.22 × 0.55 = 0.1210	0.1210/0.2713 = 0.45
C	0.67	0.18	0.67 × 0.18 = 0.1206	0.1206/0.2713 = 0.44
			0.2713	1.00

The “news” event in this illustration has resulted in making Events B and C approximately equally likely based on all information available to the DM.

Conclusions

When assessing probabilities over several events, estimating each event’s probability in turn is a flawed process. Probabilities of events other than the one being assessed may be considered but only as a group probability if at all. The final event assessed is in effect a “residual” probability as in one minus the sum of the probabilities of the previously considered events.

The pairwise comparison procedure outlined above more specifically compares respective likelihoods of events two at a time and uses comparisons against at least two other events in most cases (the least and most likely events excepted). And in a complete AHP analysis, each event is compared with every other possible event. Any difficulties as with events with widely dispersed likelihoods can be circumvented with a reduced number of comparisons over all events as outlined above. In both cases, several pairwise comparisons determine the initial probability distribution and no probability is determined as a “residual” probability. This initial probability distribution can then be adjusted to reflect the ultimate judgments of the DM on the relative likelihoods of all possible events based on a holistic view of the probability distribution now determined. For example, the probability of A above could be reduced to 0.1 and that for Event C could be raised to 0.45 making it equally probable with Event B. With limited background information as for the Lehman failure, differentiating probabilities between two scenarios or events by just 1% risks a charge of spurious accuracy.

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