THINKING PROBABILISTICALLY

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Probability distributions are derived for real-world situations involving hard-to-quantify outcomes unlike drug trials for example. Scenarios may involve levels of satisfaction, risk, inflation etc. that show extreme volatility over time requiring frequent updating. Pairwise judgments by the decision-maker form the basis for the simple calculations that could replace traditional revision of prior distributions.

Keywords: probability assessment, Bayesian revision, pairwise judgments, reciprocal matrices

Introduction

Thinking probabilistically as advocated by Nassim Taleb ("considering that alternative outcomes could have taken place, that the world could have been different, is the core of probabilistic thinking", Taleb (2005)), is not currently normal practice when decision makers (DMs) select an action in the face of uncertainty. Current practice favours an action based on viewing the most likely outcome as *certain* to happen, or worse, not even considering other possibilities. Before deciding on the action to implement, it could be advantageous to estimate the probabilities of all possible outcomes (suitably defined to be complete or exhaustive) and then consider their relative likelihoods. If one scenario or outcome has a very high probability, there would be more confidence in the resulting action that is taken as being "best" in the circumstances. Conversely, if no scenario dominates the likelihoods, it may be better to postpone acting until the situation becomes clearer, if this is possible. But thinking probabilistically necessarily involves probability assessments and currently this is no easy task. Current practice favours estimating the probability of each outcome separately in a sequential fashion. This means the probability of the last outcome considered is predetermined as one minus the probability sum of all preceding outcomes or scenarios. That is, the-sum-to-unity axiom means reassessment may be required until it is satisfied. If this happens, probabilities thought to be "correct" initially by the DM must be revised to satisfy the axiom.

Rationale for Probabilistic Thinking

Mohamed El-Erian (2016) outlines how probabilistic thinking helped PIMCO (a fixed fund manager) react quickly to the Lehman Brothers collapse in 2008. The fund envisaged three possible scenarios: a takeover by a stronger bank as happened with J.P. Morgan's takeover of Bear Sterns six months earlier (highest probability), an orderly liquidation similar to the Long Term Capital Management wind-up in 1998, or a disorderly liquidation (lowest probability). The lowest probability event did in fact materialize and in El-Erian's words: "While we had gotten the probabilities wrong, the preemptive analysis and the associated action plans had enabled us to quickly get back onside", (p. 241, 2016). Conversely, other financial institutions were blindsided by the unexpected outcome and ended up at risk from the resulting fall-out in financial markets. Of course, low probability events manifest themselves on occasion notwithstanding El-Erian's comment on getting the probabilities "wrong".

Structuring Probability Assessments

Hughes (2020) outlines a simple procedure for deriving probabilities for all outcomes in an n-event problem. This involves (n-1) relative pairwise judgments after ordering the outcomes from least to most likely. Some outcomes could show equal likelihood. The least likely outcome is assigned a base value of 1.0 followed by a pairwise judgment as to how much greater likelihood (more likely) the second event in the ordering is over the least likely event. A series of such pairwise judgments are made in sequence up to the most likely event. At any point in the ordering, equal likelihood of outcomes for two events is denoted by 1.0, otherwise all values will be greater than 1.0 indicating greater likelihood for the more favoured event. Pairwise judgments are discussed in more detail in Saaty (1980; 2005).

Using the Lehman Brothers failure as an example, three possible outcomes can be detailed in order of increasing likelihood as noted by El-Erian:

1. A disorderly liquidation or DL (least likely base value at 1.0).
2. An orderly liquidation or OL (between 2 and 3 times more likely than DL).
3. A takeover by a stronger bank or TO (almost twice as likely as OL).

The above pairwise judgments are not those of PIMCO but simply serve to illustrate the procedure. Estimated probabilities are derived in Table 1.

Table 1

Estimated Probabilities of Lehman Failure Based on Pairwise Judgments

Scenario	Pairwise Value	Compound Value	Probability	Percent Probability
DL	1.00	1.00	1.00/8.125 = 0.123	12%
OL	2.50	1.00 x 2.50 = 2.50	2.50/8.125 = 0.308	31%
TO	1.85	2.50 x 1.85 = 4.625	4.625/8.125 = 0.569	57%
		8.125	**1.000**	**100%**

Note that TO is judged 85% more likely than OL which is a little less than twice as likely as the original judgment stated. The OL pairwise judgment is between 2 and 3 times more likely than DL as stated originally. These pairwise values are only illustrative. To avoid any criticism of spurious accuracy, the DM could report the probabilities as 10%, 30% and 60% respectively. The corresponding more likely values are then 1, 3 and 2 which are in the same "ballpark" as the original pairwise judgments above.

The pairwise or primary judgments will determine the "ballpark" probability distribution over all possible outcomes after simple calculations are made as above. These primary judgments may be vague, rudimentary or tentative. For example, using 1.5 for a pairwise comparison of two events where the second event does have greater likelihood but is not twice as likely in the DM's judgment. Similar initial judgments will define the "ballpark" of all probabilities for outcomes in the situation facing the DM. Within this "ballpark" the DM can settle on a final probability distribution by altering these initial results suitably. The advantage of this approach over the current, direct sequential estimation procedure is that a complete probability distribution is presented to the DM initially that satisfies all probability axioms. And final judgments can then be made using other information. For example, the probability that two particular events must sum to less than 20% in the DM's thinking, or similar such judgments. That is, the "ballpark" distribution simply sets the stage before determination of a final distribution by the DM. In contrast, the current sequential procedure means that the reassessment can only go one way. For example, probabilities that sum to greater than 1.0 must be reduced from what was initially considered correct to satisfy the sum-to-unity axiom (or increased if too small initially). That is, individual outcomes are not re-assessed on their likelihood merits but on the need to satisfy an axiom. The current practice of assessing probabilities sequentially may be facilitated after deriving the "ballpark" distribution using the above procedures.

Rather than use the minimal (n-1) calculations above, every outcome or event could be pairwise compared with every other possibility resulting in an (n x n) reciprocal matrix for analysis. This involves n(n-1)/2 pairwise judgments. In this case, the principal eigenvector method of Saaty (1980) or the geometric mean methodology of Crawford (1987) can be employed to derive the probabilities.

Using simple calculations, a perfectly consistent reciprocal matrix can be formed from the minimal (n-1) judgments as explained in Hughes (2020). Then, after appraisal by the DM, some or all of the non-adjacent pairwise values can be replaced by more appropriate values in the DM's judgment. This results in a "ballpark" distribution based on more information than utilized by the minimal (n–1) judgments. A non-adjacent pairwise value consistent with the previous judgments may trigger a more considered appraisal by the DM taking into account factors possibly overlooked when deciding on the preceding values. For example, using the above judgments from Table 1, a perfectly consistent value of 2.5 x 1.85 or 4.625 would be in the (3,1) position of a 3 x 3 reciprocal matrix and 1/4.625 in the (1,3) position. This results in the same probabilities as shown in Table 1 using either the principal eigenvector or geometric mean methodologies. If, however, the DM now decides that the more likely TO/DL value should be 4, the probability distribution becomes 0.132, 0.314 and 0.554 for DL, OL and TO, respectively. This revised judgment, making the TO/DL more likely value a little less than the OL/DL and TO/OL judgments imply, (4 < 4.625), slightly raises the probabilities of DL and OL and lowers that for TO. Alternatively, a more likely TO/DL value of 5 results in the distribution 0.119, 0.304 and 0.577, respectively and does the reverse.

Bayesian Revision

The probabilities derived above can be considered as prior probabilities in Bayesian terminology as in McGrayne (2011). New information can then be used to update these probabilities into posterior probabilities using Bayes Theorem. Again, the mechanics of this procedure are outlined in Hughes (2020). The essential point is that each outcome postulated is evaluated pairwise as to its *consistency with the new information*. That is, all outcomes are ranked in order from least to most consistent with the new information and then pairwise compared as above using 1.0 for a pairing that is equally consistent. In some cases, an outcome could have zero consistency with the new information. For example, in a candidate selection process, new information may come to hand that a woman will be selected. In this case, all male candidates have zero consistency with the new information and the posterior will assign them a zero probability in the updated distribution.

Experimental situations such as drug trials, lend themselves to clearly defined inputs and outputs (e.g., drug or placebo, success or failure etc.). Use of Bayesian priors and posteriors follows naturally in these situations. Real-world problems involving family, financial and other day-to-day activities may involve more nuanced assessments such as low, medium or high levels of satisfaction, risk, difficulty etc. And the internet world with constant innovation and news dissemination (often distorted or even misinformation) can make for continual and sometimes radical revisions in judgments. This can result in fundamental rather than incremental changes in assessments. In these situations, starting afresh at each point in time with a minimal number of pairwise values as outlined here may be more practical than deriving a traditional Bayesian posterior incorporating previous and possibly now outdated prior judgments.

Conclusions

Probabilistic thinking should be encouraged if only to advance El-Erian's point that it promotes thinking about associated actions following whatever outcome does in fact eventuate. For the DM, there are no surprises apart from a low probability event manifesting itself in given situations.

Only when a complete probability distribution is appraised do certain attributes become apparent. For example, a distribution may be weighted too heavily in favour of good (or bad) outcomes. Readjustments can then be made in the light of the complete picture.

A probability distribution can only be "correct" if it accurately reflects the considered judgments (or degree of belief) of the DM. By definition, the final distribution of the DM must be correct since he or she must always agree with the final values determined or would change them otherwise. At the very least, the final values are close enough for any time-constrained decision making.

Good judgment in chance situations comes down to: What was the probability of the outcome that eventuated? Consistently high probabilities assigned to resulting outcomes means the DM possesses good judgment. Even if a low probability event does occur, the DM has presumably thought ahead of time what he or she would do in that eventuality and advance his/her situation over any rivals who did not do so.

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