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A New Approach to Probability Assessment

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Using spreadsheets and ranges for pairwise judgments, candidate probability distributions are generated for the decision-maker to consider. This replaces event-by-event determination of probabilities. Basic statistics of the distributions are then used to determine a final distribution for decision purposes as in buy, sell, or hold.

Keywords: probability assessment, pairwise judgments, spreadsheet analysis, statistical measures

Introduction

Modern computers and software such as spreadsheets have facilitated new approaches to probability assessment. In this short review, a methodology is outlined whereby the decision-maker (DM) uses pairwise assessments of possible events to generate resulting axiomatically correct probability distributions. These distributions comprise the "ballpark" within which the DM determines a final distribution possibly using other information. This replaces the current event-by-event determination of probabilities which may need repeated reassessment before probability axioms are satisfied.

Typically, real-world problems involve a modest number of discrete events such as a small price, quantity etc., change (±), a large change (±), or no change—a total of five events in this case. First the events are ordered from least to most likely; then "more likely" judgments are made by the DM with 1.0 denoting equally likely, 1.25 a little "more likely", 2 for twice as likely and so on. Suggested pairwise values are discussed more fully in Hughes (2020). These "more likely" judgments are the basic primary inputs that determine the resulting probability distribution of the DM.

Illustrative Example

The procedures are most easily explained via an example. To illustrate, we take four possible events A, B, C, and D arranged in order of increasing likelihood. The DM assesses the relative likelihoods as summarized in Table 1.

Table 1
Possible Pairwise Values for the Four Events

	Pairwise values	
Event	Low	High
A (Base 1)	1.00	1.00
B/A	2.00	3.00
C/B	1.50	2.00
D/C	1.25	1.80

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Table 1 shows the B/A likelihood ratio to be between two and three times "more likely" for the favored B event. The D/C value shows D is judged to be between 25% and 80% "more likely" than C.

All subsequent calculations emanate from the six "more likely" values in Table 1. Equally likely events would use 1.0 in both columns. The "more likely" ranges for the events determine 2^3 (more generally $2^{(n-1)}$ for n events) or eight possible distributions using appropriate combinations of the "more likely" values. Pairwise assessment (based on much experience) is outlined in Saaty (2008), with the B/A value ultimately determining the numerical priority (in Saaty's terminology) that B has over A. Priorities become probabilities here. Table 2 summarizes the calculations needed to determine the probability distribution for the Low column in Table 1.

Table 2

Probability Distribution Determination From Pairwise Judgments

Event	Pairwise value	Compound likelihood	Probability
A	1.00	Base 1.00	1.00/9.75 = 0.1026
B	2.00	1.00 x 2.00 = 2.00	2.00/9.75 = 0.2051
C	1.50	2.00 x 1.50 = 3.00	3.00/9.75 = 0.3077
D	1.25	3.00 x 1.25 = 3.75	3.75/9.75 = 0.3846
Totals		9.75	1.0000

Note that the D/A pairwise value calculated as (B/A)(C/B)(D/C) is 3.75. If the DM wished to directly estimate the D/A value (and other non-adjacent event pairwise judgments), this can be accommodated in the Saaty methodology but involves the use of eigenvectors, geometric means, or other averaging techniques to determine the final distribution. A modern summary of possible averaging alternatives is outlined in Brunelli (2015). Some examples using these techniques are presented in Hughes (2022).

All combinations of the pairwise values lead to eight possible distributions as shown in Table 3 and are routinely calculated in a spreadsheet. The Table 2 probabilities are Distribution 1 in Table 3.

Table 3

Resulting Probability Distributions Over Events A, B, C, and D From the Pairwise Values in Table 1

Event	Candidate probability distributions from initial pairwise judgments							
	1	2	3	4	5	6	7	8
A	0.1026	0.0877	0.0833	0.0704	0.0708	0.0602	0.0571	0.0481
B	0.2051	0.1754	0.1667	0.1409	0.2124	0.1807	0.1714	0.1442
C	0.3077	0.2632	0.3333	0.2817	0.3186	0.2711	0.3429	0.2885
D	0.3846	0.4737	0.4167	0.5070	0.3982	0.4880	0.4286	0.5192
Sum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Distribution 8 in Table 3 uses the pairwise values in the High column of Table 1. Candidate probability distributions in Table 3 constitute the "ballpark" with statistical analysis in Table 4 using standard mean, median, and range measures. The DM can then input his/her final judgments in the Percent column for decision purposes. Currently, Table 4 shows the rounded Average probability as Percent.

Table 4

Statistics on the Candidate Probability Distributions in Table 3 for Events A, B, C, and D

	Probabilities		Range statistics				Probabilities		More likely value
Event	Mean	Median	Low	High	Midpoint	Spread	Average	Percent	
A	0.0725	0.0706	0.0481	0.1026	0.0753	0.0545	0.0728	7	Base = 1.00
B	0.1746	0.1734	0.1409	0.2124	0.1766	0.0715	0.1749	18	18/7 = 2.57
C	0.3009	0.2981	0.2632	0.3429	0.3030	0.0797	0.3006	30	30/18 = 1.67
D	0.4520	0.4511	0.3846	0.5192	0.4519	0.1346	0.4517	45	45/30 = 1.50
Sum	1.0000	0.9932	0.8368	1.1771	1.0068	0.3403	1.0000	100	
				Average spread = 0.0851					

The values under Average in Table 4 are an average of the Mean, Median, and Midpoint values. The last column in Table 4 shows the "more likely" values based on the DM's percentage probabilities in the preceding column. So, the final D/C value at 1.5 here is within the required range as in Table 1 of 1.25-1.80 for this pairwise value. The original pairwise values used as in Table 1, however, are not set in stone. They may only be tentative or vague and the DM's thinking may evolve during the investigation with the resulting pairwise values for the final distribution outside the ranges of the initial inputs as in Table 1. Of course, with the ease of spreadsheet calculation, revised pairwise value ranges can be employed at any time with resulting candidate distributions and statistics routinely re-calculated.

Note that the Mean, Median, and Range Midpoint values in Table 4 are all closely aligned for each event differing by at most 1%. It may be that, based on other judgments (e.g., one of A, B, or C has a 50% chance), the DM (after some reflection) may use a distribution closer to number 8 in Table 3. For example, 5%, 15%, 30%, and 50%, with pairwise values 1, 3, 2, and 1.67. Probabilities could then be validated (or re-calculated) using these (or similar) "more likely" values.

Conclusions

The above procedures are not demanding of the DM (initial pairwise ranges) and produce axiomatically correct probability distributions for the DM's consideration. Comparing distributions may be easier than event-by-event calculations. Modern problems confronting DMs (e.g., the origin of COVID-19) are not so clear-cut as drug trials or polls of constituents where relative frequencies on clearly defined outcomes are readily available. Typically, in the real-world situations experienced today, nuanced judgments are required and, even if such judgments are not precise, they can be managed as in the preceding analysis.

The probability of an outcome or event is in the mind of the DM. Unlike relative frequencies, the degree of belief on the likelihood of a given outcome is peculiar to that DM. By quantifying qualitative beliefs, the DM succinctly summarizes by way of probability a path to making a decision as in buy, sell or hold.

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