Chinese Business Review, Jan. 2018, Vol. 17, No. 1, 33-37
doi: 10.17265/1537-1506/2018.01.003

Assessing Probabilities of Unique Events in Decision Making

Warren Richard Hughes
The University of Waikato, Hamilton, New Zealand

This paper outlines a two-stage assessment procedure for deriving probabilities of unique events. The calculations are illustrated using possible events pertaining to the Lehman failure in 2008. The procedures utilize pairwise comparisons associated with the Analytic Hierarchy Process. Typical betting odds are used to motivate an ordering of qualitative judgments that are then converted into quantitative assessments and finally a probability distribution.

Keywords: probabilities, betting odds, Analytic Hierarchy Process, Lehman failure, Bayesian Revision

Introduction

It is common for a decision maker (DM) to find himself/herself faced with a decision problem where one of several events could occur and influence the payoff from implementing one of the alternative actions available. Once probabilities are assigned to the possible events, traditional decision theory tools such as expected value, value of information etc. can be employed to aid in the decision process leading to a chosen action to undertake.

Probability determination over the event space can be difficult where the situation is unique to the DM and probabilities must be subjectively assigned by the DM. Typically, no "objective" past history (or very little) is available to the DM although he/she may have ideas as to the relative likelihoods of the possible events (primary judgments). Calibrating any such vague, intuitive, personal, and qualitative judgments into a quantitative probability distribution over the set of discrete mutually incompatible events is examined below.

The Analytic Hierarchy Process (AHP)

One approach to incorporating subjective judgments about intangible quantities into decision making is that of Saaty's AHP (1980; 2005). AHP is a methodology to quantify judgments about various elements as to whether one element is better, longer, heavier, more likely etc. than another element and by how much when "objective" measurement is not available. Pairwise comparisons are made between all elements with a resulting index ranking all elements from best to worst with a quantitative value associated with each element, defined as priorities by Saaty. Brunelli (2015) provides a modern summary of AHP procedures.

For probability assessment, the relevant property of each event (element) is "more likely" relative to some alternative event in the event space of all possible and mutually incompatible events. Any resulting probability distribution over all such events is by definition a measure of their relative likelihood or occurrence. In this respect, use of AHP in probability derivation, as a theory and methodology of relative measurement, seems a natural approach in proceeding with the assessments.

Warren Richard Hughes, Dr., The University of Waikato, Hamilton, New Zealand.
Correspondence concerning this article should be addressed to Warren Richard Hughes, 6 Taurarua Terrace, Parnell, Auckland 1052, New Zealand.

Betting Odds

Traditionally, betting odds express probabilities in terms of money wagered and won if a particular event occurs. Sporting events typically engender betting odds set by bookmakers but other situations can be the subject of odds setting (e.g. elections or similar situations). For example, a 3:1 bet (against) pays $3 if the event occurs for a $1 wager which is lost if the specified event does not occur. The implied probability of the event occurring is 1/(3 + 1) or 0.25. Alternatively, a 3:1 bet implies that the situation of the bet could be re-run or trialled four times with the specified event occurring once in the four trials.

Sporting events typically have a history of outcomes that enables bookmakers to set odds based on past performances under similar conditions to the present. Unique events, however, have no such history (or very little) so setting "objective" odds does not seem feasible in these situations. Quoted betting odds, however, can be used to suggest an alternative procedure. For example, a 3:1 candidate event is 25% "more likely" than a 4:1 candidate as in 5/4 or 1.25. And a 4:1 candidate is twice as likely as a 9:1 candidate since the first candidate occurs once in five trials whereas the second requires 10 trials for one occurrence (10/5).

If the DM can make qualitative pairwise comparisons on a more likely basis, the following classification scheme is suggested.

Table 1

Qualitative to Quantitative Assessment

Pairwise qualitative assessment for two events	Range of possible values	Mid-point
Equally likely event	1.00	1.00
Slightly more likely event	1.00^{+}-1.20^{-}	1.10
Significantly more likely event	1.20-1.80^{-}	1.50
Substantially more likely event	1.80-2.00^{-}	1.90
Twice as likely event	2.00	2.00

The modifiers slightly, significantly, and substantially may represent different magnitudes to different people but remain consistent for a single DM over a set of assessments. Once the order of magnitude is determined, as in less than twice as likely, then a value in the range corresponding to the appropriate modifier is selected. The same procedure applies for higher orders of magnitude as in 2.5 signifying an event is significantly more than twice as likely as an alternative. Alternatively, a value of 2.5 could reflect an imprecise judgment that one event is between two and three times more likely than another event.

The value-attribution procedure is easier to apply than trying to calculate explicit odds for a candidate event occurring. First, the events are ordered from least likely to most likely. Then, a pairwise comparison results in an appropriate more likely value for the candidate event compared to an event of adjacent but lesser likelihood. After all such pairwise assessments, subsequent calculations as outlined below result in a probability distribution over all events.

Analysis of the Lehman Failure in 2008

In his book *The Only Game in Town*, Mohamed A. El-Erian (2016) outlined how PIMCO (a fixed income investment manager) approached the looming Lehman crisis in September 2008. PIMCO postulated three possible scenarios for a Lehman future:

(1) Lehman is taken over by a stronger bank such as Barclays (scenario or event TO).

(2) Lehman follows an orderly liquidation similar to that of the hedge fund Long-Term Capital Management in 1998 (OL).

(3) Lehman goes into a disorderly liquidation (DL).

Anticipating that Lehman would follow the same scenario as that of Bear Sterns with its takeover by J. P. Morgan in March 2008, PIMCO assigned the highest probability to the TO scenario and the lowest to DL. El-Erian does not state what the probabilities were other than to say the probability of TO was “rather high”. It turned out that the least likely scenario of DL was the one that eventuated.

The methodology outlined here is used to derive some illustrative probabilities using the PIMCO ordering of the scenarios.

Table 2

Stage 1: Derivation of Probabilities on the Lehman Failure

Scenario or event	Event ordering	More likely pairwise comparison	Compound value	Stage 1 probabilities
DL	Least likely	1.00	1.00	1.00/6.16 = 0.16
OL	Intermediate likelihood	1.75	1.75	1.75/6.16 = 0.28
TO	Most likely	1.95	3.41	3.41/6.16 = 0.56
Total			6.16	1.00

Note that the pairwise value for the first or least likely event is defined as 1.0. The above values are illustrative only and not related to any analysis by PIMCO. Event OL is significantly more likely than DL and event TO is substantially more likely than OL. In fact, TO is just less than twice as likely as OL.

For intermediate events like OL (excludes the most and least likely events), there are pairwise comparisons with two other events providing some diversity in establishing its relative likelihood.

For the OL pairwise value determination, DL is the base event and OL the more likely adjacent event. A second related procedure would be to use a “less likely” comparison with OL as the base event and DL then evaluated as to its lesser likelihood. In this case (Stage 2), we order the events starting with the most likely event TO with a defined 100% value. The calculations are summarized in Table 3.

Table 3

Stage 2: Derivation of Probabilities on the Lehman Failure

Scenario or event	Less likely pairwise value	Equivalent more likely value	Compound value	Probabilities		
				Stage 1	Stage 2	Average or final
TO	100%	100/40 = 2.50	4.55	0.56	0.62	0.59
OL	40%	100/55 = 1.82	1.82	0.28	0.24	0.26
DL	55%	1.00	1.00	0.16	0.14	0.15
Total			7.37	1.00	1.00	1.00

Table 3 shows that event or scenario DL has 55% of the likelihood of OL in the Stage 2 assessments. Alternatively, OL has a more likely value of 100/55 or 1.82 relative to DL. This compares with the 1.75 value at Stage 1. Judgments at either Stage reflect a different orientation and should not be expected to be perfectly consistent with each other. Although a two-stage process may seem somewhat redundant, it does guard against a single mis-judgment from adulterating the resulting probability distribution. Any significant inconsistencies could be resolved at either stage and the probabilities re-worked.

Probabilities are shown to two decimals as an acceptable level of precision based on intuitive or imprecise qualitative judgments as to relative likelihoods. From Table 3 we see that TO is 0.59/0.15 or 3.93 times more likely than DL. This particular pairwise comparison was not made directly as would be the case with a complete AHP procedure. A precise evaluation such as 3.93 in real-world practice seems very unlikely with 4 a credible (and reasonably consistent) assessment. This raises the fundamental question, however, as to how a DM can plausibly make such judgments on relative likelihoods between the most likely outcome and the least likely outcome. Especially if these likelihoods are extremely diverse as may be the case. For example, if the probability of TO was 85% and that for DL 5%, the TO assessment becomes 17 times more likely than DL. Rather than have such judgments on diverse events that may possibly distort the probability derivation process, it is considered expedient to exclude them entirely. For an n-event situation, total judgments under a full AHP procedure would number n(n-1)/2 compared to 2(n-1) judgments for the methodology using the two-stage process outlined here. For n = 10 events, the number of judgments would be 45 and 18 respectively. Or just 9 if only a 1-stage assessment is employed.

Bayesian Revision

New information on the situation under analysis may require revision of the probability distribution (the prior) derived above. In this case, the methodology outlined above can still be utilized. For the revision, the more or less “likely” criterion is replaced by a more or less “consistent with the new information” criterion. Events having zero consistency with the new information can be eliminated. At Stage 1, the event(s) having least consistency with the new information is (are) determined and other events ranked sequentially up to the event(s) most consistent with the new information. Of course, this order may be completely different from the ordering used for the determination of the prior or initial probability distribution. A Stage 2 derivation of likelihood using less likely assessments follows the same procedure as above for the prior distribution with an appropriate ordering of events from most to least consistent with the new information.

Probabilities so determined become the likelihood in the Bayesian Revision process and the posterior distribution determined in the usual fashion as in the prior times likelihood calculations followed by normalization.

Conclusions

If the DM assesses a probability distribution by considering each possible event separately in turn, then for an n-event problem the assessed probability of the last event considered is determined as a residual probability. That is, one minus the sum of the probabilities of the previously assessed n-1 events. This means the primary likelihood of this “residual” event plays no direct role in deriving an initial or resulting probability distribution. If the probability of this “residual” event is then adjusted to better accord with the DM’s primary judgments on its likelihood, another of the n-1 events becomes the “residual” event, and so on. For the methodology outlined here, the primary judgments on the likelihoods of all possible events play a role in determining the initial or resulting probability distribution at Stage 1 and 2.

A DM could derive only the Stage 1 (or Stage 2) probabilities and then adjust the resulting probability distribution to reflect his/her final judgments on the likelihoods. Two estimates for each of the relative pairwise likelihoods may, however, mitigate the consequences of a single mis-judgment at either stage, or prompt a reconciliation of divergent primary judgments. The 2-stage process appeals as a reasonable compromise versus the full AHP evaluation which may require many more (and difficult) judgments.

While the methodology outlined here is a conventional application of the AHP, it does determine a probability distribution from primary or rudimentary judgments on relative likelihoods. Once the initial probability distribution is formulated, its realization may prompt refinements reflecting the DM's ultimate judgments on relative likelihoods.

References

Brunelli, M. (2015). *Introduction to the analytic hierarchy process*. Heidelberg: Springer.

El-Erian, M. A. (2016). *The only game in town*. New York: Random House.

Saaty, T. L. (1980). *The analytic hierarchy process.* New York: McGraw-Hill.

Saaty, T. L. (2005). *Theory and applications of the analytic network process.* Pittsburgh: RWS Publications.