

PROBABILITY ASSESSMENT

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The methodology presented below can be viewed as a means of quantifying intuitions, guesses, hunches etc., about relative likelihoods for alternative events leading to a “ballpark” distribution. A final distribution can then be formulated by the decision-maker using other information as in minimum or maximum collective probabilities for groups of events or similar assessments. Final judgments may be idiosyncratic to the decision-maker and not easily replicable in an algorithm.

Keywords: probability assessment, pairwise judgments, spreadsheets, minimal calculation

Introduction

Thinking probabilistically was advocated by Nassim Taleb: “considering that alternative outcomes could have taken place, that the world could have been different, is the core of probabilistic thinking”, Taleb (2005). It was evidenced in the Lehman failure of 2008. PIMCO (a fixed fund manager) postulated at the time three possible scenarios. In order of increasing likelihood these were: a disorderly liquidation, an orderly liquidation (as for Long-Term Capital Management) and finally a takeover by a stronger bank. In his book, PIMCO executive El-Erian (2016) characterized the occurrence of the lowest probability outcome as follows: “while we had gotten the probabilities wrong, the preemptive analysis and associated action plans had enabled us to quickly get back onside”. This illustrates the benefits of thinking probabilistically. All that remains is an easy methodology for calculating probabilities which is the aim of this note.

Methodology

Moderate numeracy skills and spreadsheet familiarity are all that are needed to undertake initial calculations leading to an axiomatically correct “ballpark” distribution over the events in the situation being considered by the decision-maker (DM). This distribution can then be fine-tuned with other judgments or information. That is, an initial distribution is formed after which the current practice of event-by-event probability assessments can then be made with a complete if tentative distribution in the background.

When considering probabilities of events, DMs have instinctive judgments as to:

- The order of the events from the least to the most likely event.
- The order-of-magnitude as to likelihood differences between adjacent events in the ordering.

A procedure to exploit these intrinsic but tentative judgments can then be employed that leads to an initial “ballpark” distribution which can then be fine-tuned with other judgments as applicable.

Examples of how such likelihood judgments between adjacent events may be made (i.e., pairwise judgments as in Saaty (2008)) with numerical equivalences necessary for the methodology are presented in Table 1. Note the events are first ordered from least to most likely as in A, B, ... , etc. Then a numerical equivalent is expressed as a ratio between the likelihood of the more likely event over the less likely event in the ordering as represented by B/A, C/B, ... etc.

Table 1: Value Equivalents for Pairwise Judgments on Adjacent Events in the Ordering

Example Judgments on Adjacent Events in Likelihood Ordering	Numerical Equivalence or Range
Events are equally likely	1.00
Event B is slightly more likely than A with a B/A value in the stated range	1.10 – 1.30
Not quite twice as likely	1.75 – 1.95
More than double but less than 3 times more likely	2.00 ⁺ – 3.00 ⁻
Much more likely	4.00 – 6.00
Extremely more likely	10.00 – 12.00

Note that a high range value for B/A involving a value of 10 or more would indicate that A is an extremely unlikely event. The subsequent C/B range value (and succeeding range values) will play an important role in determining B’s relative likelihood. A high range value at any point in the ordering means the probabilities of the denominator event and all preceding events in the ordering will be low. Note a range could be determined by considering a value such as 2.5 initially and then utilizing an interval as in 2 – 3. Thinking of a single value initially, as in order-of-magnitude, and then expanding to an encompassing range may be a good strategy. Furthermore, spreadsheets allow “ballpark” distributions to be easily recalculated if initial results are significantly out-of-line with the DM’s thinking. For example, an extremely more likely range of 10 – 12 may be halved to 5 – 6 and the distribution recalculated using this lower value range.

Once the implications of these “tentative” range judgments are evidenced in an actual distribution, the DM can refine the probabilities to reflect his/her more considered judgments and/or other available information.

Illustrative Example

The methodology is best illustrated by example. Table 2 details a 5-event problem with illustrative events A, B, C, D, E and associated pairwise ranges. Note that the B/A range is 4 – 6, indicating a low probability for event A. Events C and D are seen as equally likely in this illustration with a D/C ratio of 1.0.

Table 2: Illustrative Calculations for a 5-Event Problem

Scenarios		Pairwise Ranges		Probabilities					More Likely Value
Events	Ratios	Low	High	Mean	Median	Midpoint	Average	Percent	
A	Base	1.00	1.00	0.02163	0.02073	0.02334	0.02190	2	1.00
B	B/A	4.00	6.00	0.10393	0.10192	0.10613	0.10399	10	4.75
C	C/B	1.50	2.50	0.19644	0.19491	0.19765	0.19633	20	1.89
D	D/C	1.00	1.00	0.19644	0.19491	0.19765	0.19633	20	1.00
E	E/D	2.00	3.00	0.48154	0.48102	0.48129	0.48128	48	2.45
				0.99998	0.99349	1.00606	0.99983	100	

A 5-event problem makes for $2^{(5-1)}$ or 16 possible distributions. Statistical measures on the resulting event probabilities are summarized in Table 2. The Midpoint value shows the average of the minimum and maximum probabilities for each event over the 16 distributions. Average is the mean of the preceding 3 values in the table. The More Likely Value shows the pairwise value using the Average Probabilities as in $0.10399/0.0219$ or 4.75. Probabilities could be validated using these pairwise values. Alternatively, a new “ballpark” distribution could be formulated using revised, single pairwise values now thought more appropriate by the DM after consideration of the Table 2 results. These calculations are outlined in Hughes (2022) and can be easily replicated in a spreadsheet.

As the number of events increases, the number of possible distributions doubles with each additional event. An alternative “ballpark” distribution can be derived using only the low and high pairwise values initially and then

averaging the resulting probabilities. These minimal calculations are outlined in Table 3 using the same judgments as in Table 2.

Table 3: Alternative Probability Calculations Using only Low and High Pairwise Values

Scenarios		Pairwise Ranges		Probabilities				More Likely Value
Events	Ratios	Low	High	Low	High	Average	Percent	
A	Base	1.00	1.00	0.03448	0.01220	0.02334	2	1.00
B	B/A	4.00	6.00	0.13793	0.07317	0.10555	10	4.52
C	C/B	1.50	2.50	0.20690	0.18293	0.19491	20	1.85
D	D/C	1.00	1.00	0.20690	0.18293	0.19491	20	1.00
E	E/D	2.00	3.00	0.41379	0.54878	0.48129	48	2.47
				1.00000	1.00001	1.00000	100	

Although there are very slight differences in probabilities using the alternative approach, percent probabilities are identical, or differ by at most one percentage point. Percent probabilities should suffice for most routine decision making. The spread between lowest and highest probabilities for each event in Table 3 may also be instructive for final decisions on appropriate pairwise ranges as will the resulting more likely values in the last column.

Conclusions

Probability determination in the mind of the DM processes information in a way that may not be easy to replicate in an algorithm. In this new approach above, the DM summarizes his/her processing with a low to high range of “more likely” values in comparing two events (pairwise values). Candidate distributions can then be calculated via a spreadsheet followed by routine methodology to axiomatically correct probability determination. The resulting “ballpark” distribution could be utilized immediately or further developed using other information possibly triggered by the analysis completed to date. With this perspective, probability determination may be better thought of as a process rather than a methodology which in one pass of calculation produces the final distribution.

Even with a large number of possible events, the above illustrates a practical methodology for probability assessment usable by anyone wanting to “think probabilistically” when making decisions. Although the methodology may not yield a definitive probability distribution in a first pass of calculation, it does go some way in quantifying initial beliefs on relative likelihoods with the “ballpark” distribution thus setting the scene for ultimate determination of probabilities.

References

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